

Masterclass: Noncommutative Index Theory

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Syllabus

1. Differential operators The algebra of differential operators, order of a differential operator, differential operators on manifolds, the principal symbol, ellipticity, examples from complex analysis. [NH]
2. Dirac operators Clifford algebras, Spinors, spin and spin-c structures, the Dirac operator, index formulas, some history. [EVE]
3. Hilbert space theory of differential operators Sobolev theory, basic elliptic estimates, hypoellipticity, self-adjointness and functional calculus, the Fredholm property. [NH]
4. The Bott periodicity approach to the index of Dirac operators Proof of the index formula for Dirac operators, based on Bott Periodicity. [EVE]
5. Heat kernels, zeta functions and local methods Trace class operators, zeta functions, meromorphic continuation, heat kernel expansions, local formulas. [NH]
6. K-homology The isomorphism between geometric and analytic K-homology as a framework for index theory. [EVE]
7. Getzler's approach to the index of Dirac operators, Rescaling, Weyl's theorem on eigenvalue asymptotics, more on Clifford algebras, Getzler's rescaling method, harmonic oscillator, the local index formula. [NH]
8. General elliptic operators in K-homology: reduction to Dirac Reduction of the index problem for general elliptic operators to Dirac operators. [EVE]
9. Toeplitz operators and other hypoelliptic operators Index theory for non-elliptic operators. [EVE]
10. Perrot's approach to index theory Cyclic cocycles, pseudodifferential operators, Perrot's index theorem. [NH]

References

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3. Higson, Nigel; Roe, John. *Analytic K-homology*. Oxford Mathematical Monographs. Oxford Science Publications. Oxford University Press, Oxford, 2000. xviii+405 pp.
4. Getzler, Ezra. Pseudodifferential operators on supermanifolds and the Atiyah-Singer index theorem. *Comm. Math. Phys.* 92 (1983), no. 2, 163–178.
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6. Kato, Tosio. *Perturbation theory for linear operators*. Second edition. Grundlehren der Mathematischen Wissenschaften, Band 132. Springer-Verlag, Berlin-New York, 1976. xxi+619 pp.
7. Perrot, Denis. Pseudodifferential extension and Todd class. *Adv. Math.* 246 (2013), 265–302.
8. Roe, John. *Elliptic operators, topology and asymptotic methods*. Second edition. Pitman Research Notes in Mathematics Series, 395. Longman, Harlow, 1998. ii+209 pp.
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11. Zimmer, Robert. *Essential results of functional analysis*. Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 1990. x+157 pp.